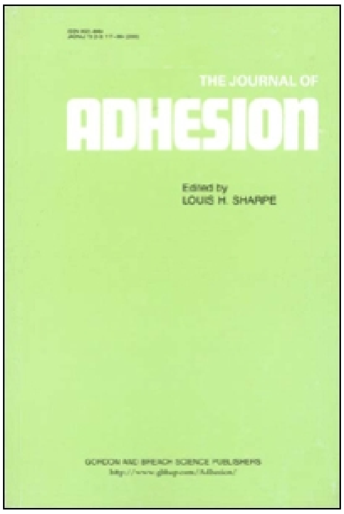

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An Analysis of the Curling Phenomenon in Viscoelastic Bimaterial Strips

Erol Sancaktar[a]
[a] Department of Mechanical and Aeronautical Engineering, Clarkson University, Potsdam, NY, U.S.A.

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An Analysis of the Curling Phenomenon in Viscoelastic Bimaterial Strips

EROL SANCAKTAR

Department of Mechanical and Aeronautical Engineering, Clarkson University, Potsdam, NY 13699, U.S.A.

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A mathematical analysis is performed to obtain relations for the radius of curvature and flexural moments for initially stretched bimaterial strips in which at least one of the materials exhibits viscoelastic behavior. One practical application of this analysis is for pressure sensitive tapes. Consequently, the radius of curvature and flexural moment relations are obtained as functions of backing and adhesive thicknesses and moduli for typical pressure sensitive tapes. The analysis shows that the flexural moment decreases as the backing thickness and/or backing modulus increase. Furthermore, the flexural moment decreases as the adhesive thickness and/or adhesive modulus decreases.

KEY WORDS pressure sensitive adhesive tapes; curling; flagging; self-peeling; strain mismatch; recovery strains; flexural moment.

INTRODUCTION

Curling or flagging are terms which describe self-peeling and lifting of the ends of a pressure sensitive tape which was initially adhering to a substrate (Figure 1). The self-peeling occurs due to the asymmetric recovery of initial stretch induced in the adhesive tape during its processing and/or its application to a substrate. After self-peeling, lifting tape forms a flag or tab tangent to the contour of the wrap, or a spiral wrapping of the tape becomes partially or completely unwound.

Consequently, self-peeling of adhesive tapes can be attributed to elastic recovery and delayed elasticity induced by residual stresses (caused by processing and/or application conditions), application pretension or other stresses induced during application, such as peeling of release liner, unwinding of tape from its core and tearing off of a length of tape.

Recovery strains induced in this manner cause unequal flexural stresses at the top and bottom surfaces of the composite tape and, consequently, result in a net peeling moment. The asymmetry in the flexural stresses is due to unequal moduli of elasticity and thicknesses of the backing and the adhesive layer. Considering a pressure

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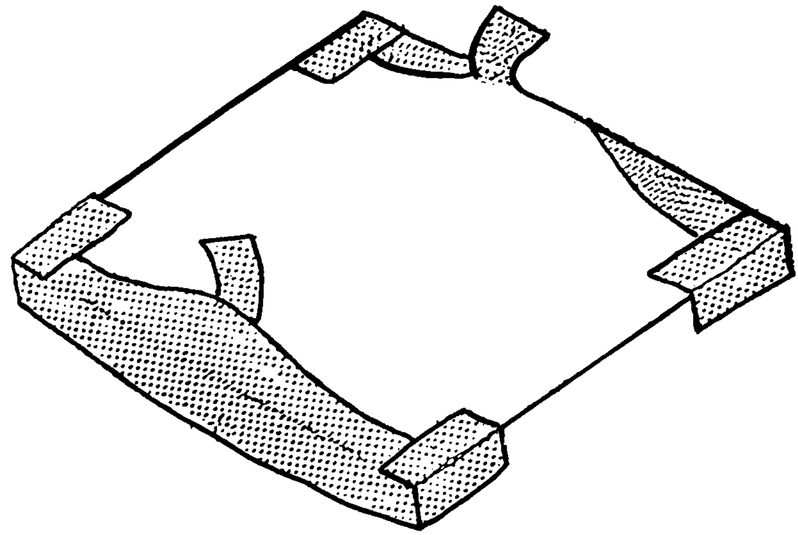

FIGURE 1 An example of curling (flagging) in pressure sensitive tapes illustrated by lifting of strip ends in the middle of taped box top.

sensitive adhesive tape adhering to a substrate, when the peeling moment created by the elastic recovery and the mismatch between the backing and the adhesive exceeds the adhesion force on the substrate, self-peeling occurs.

Elastic and linear viscoelastic approaches have been used to analyse various aspects of films, beams or plates with induced curvatures. Greener *et al.*[1] developed a phenomenological model for bending recovery in polymer films by extending the classical bending theory to linear viscoelastic materials. They used homogeneous material films but assumed dissimilar tensile and compressive bands across the film interrelated through "temporal shift." The relaxation modulus of the polymer was modeled using the generalized Maxwell function and the effect of temperature was included using the time-temperature principle. Dillard[2] obtained closed form solutions to predict the peel stresses between adherends which form a parallel bond but would have slightly different curvatures in their unbonded, stress-free states. He used an elastic approach for this purpose and proposed that for the case of bimaterial beams the inverse of the sum of the individual compliances be used to represent the effective beam stiffness. Neither analyses offered mathematical relations which explicitly included thicknesses or elastic moduli for the components of a bimaterial strip with induced curvature. Such relations will be developed in this paper.

The method of analysis to be presented for the self-peeling problem is based on the "equivalent stiffness" approach.[3] With this method an equivalent stiffness relation is developed for the adhesive (thickness t_a)/backing (thickness t_b) composite (Figure 2) so that it can be treated as a homogeneous beam under bending. This is done by (hypothetically) adjusting the width of each component in the same proportion as their moduli of elasticity make with the modulus of the homogeneous

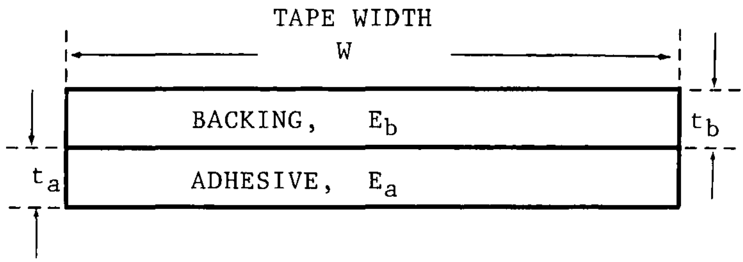

FIGURE 2 Pressure sensitive tape considered as bimaterial composite strip.

(assumed) beam (see Appendix). Consequently, the equivalent stiffness property, EI, of the strip is obtained as:

$$EI = K_1(Wt_a^3 t_b E_a E_b)/12(t_a E_a + t_b E_b) \tag{1}$$

where

$$K_1 = 4 + 6(t_b/t_a) + 4(t_b/t_a)^2 + (E_b/E_a)(t_b/t_a)^3 + (E_a/E_b)(t_a/t_b) \tag{2}$$

with E_b and E_a representing the backing and adhesive Young's moduli and W is the tape width. Derivation of Equations (1) and (2) is shown in the Appendix.

ANALYSIS FOR FLEXURAL MOMENT AND RADIUS OF CURVATURE

I—Analysis Based on the Assumption of Elastic Backing Behavior Obtained at High Rates of Loading:

Based on the geometry shown in Figure 3, standard elastic beam flexure analysis results in the following basic equations:

Flexure strain ϵ is given by:

$$\epsilon = y/\rho \tag{3}$$

where y is the distance from the neutral axis to the point of interest along the thickness of the strip and ρ is the radius of curvature created by the applied moments. Consequently, based on Hooke's law, the flexure stress can now be calculated by

$$\sigma = Ey/\rho \tag{4}$$

The moment resultant, M, on any cross section, A, of the strip can be calculated as

$$M = \int \sigma y dA = E/\rho \int y^2 dA \tag{5}$$

Since the integral on the far right side of Equation (5) defines the area moment of inertia, I, we can now write:

$$1/\rho = M/EI \tag{6}$$

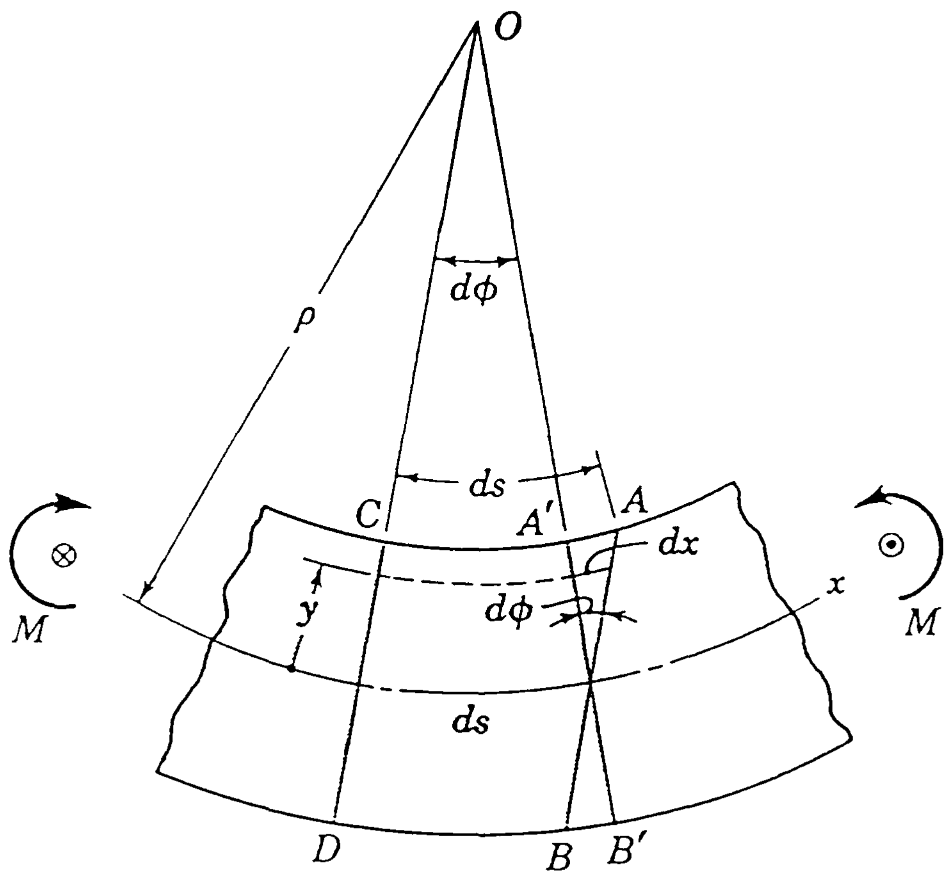

FIGURE 3 Flexure geometry.

Since a pressure sensitive tape constitutes a backing/adhesive composite strip, we need to express strain, ϵ, at any distance, y, from its backing/adhesive interface as

$$\epsilon = \epsilon_0 + (y/\rho) \tag{7}$$

where ϵ_0 is the strain at the interface and (y/ρ) is the strain due to bending of the strip. Note that ϵ_0 is common to both the backing and the adhesive and, consequently, the mechanics requirement for continuity of strains across the interphases is not violated. Also note that the space variable, y, of Equation (7) is assumed to originate at the backing/adhesive interface and is positive upward as shown in Figure 3. Consequently, calculated (see Equation 12) and measured radius of curvature values will be in reference to the backing/adhesive interface. The flexural moment values to be calculated (see Equation 13) based on Equation (6) using the equivalent stiffness Equation (1), however, will be in reference to the neutral axis (centroidal axis of the transformed section, see Appendix) as necessitated by the force equilibrium condition which is satisfied by the equivalent stiffness method.

We now define recovery strains in the backing and adhesive as ϵ_b and ϵ_a, respectively, so that the corresponding stresses become:

$$\sigma_b = E_b\,[\epsilon_o + (y/\rho) - \epsilon_b] \qquad 0 \le y \le t_b \tag{8}$$

and

$$\sigma_a = E_a\,[\epsilon_o + (y/\rho) - \epsilon_a] \qquad -t_a \le y \le 0 \tag{9}$$

In order to be able to solve the boundary value stress-strain problem for the composite strip, we need to satisfy two basic equilibrium conditions: force equilibrium and moment equilibrium. The force equilibrium dictates that

$$\int_0^{t_b} \sigma_b\, W\, dy + \int_{-t_a}^{0} \sigma_a\, W\, dy = 0 \tag{10}$$

Similarly, the moment equilibrium is obtained as:

$$\int_0^{t_b} \sigma_b\, Wy dy + \int_{-t_a}^{0} \sigma_a\, Wy dy = 0 \tag{11}$$

Integration of Equations (10) and (11) can be performed after substitution of Equations (8) and (9) for σ_b and σ_a, respectively. After integration, the interface strain, ϵ_o, can be expressed in terms of ϵ_a and ϵ_b using the integrated version of Equation (10). Subsequently, this equation (force equilibrium) can be substituted for the ϵ_o term in the integrated version of Equation (11) (moment equilibrium) to result in the radius of curvature, ρ, equation:

$$\rho = K_1/6(\epsilon_a - \epsilon_b)[(1/t_a) + (t_b/t_a^2)] \tag{12}$$

Note that if the composite strip is initially curved (prior to curling) with $(1/\rho_o)$ representing the initial curvature for the interface, then the inverse of left hand side of Equation (12) can be written as $(1/\rho) \pm (1/\rho_o)$ depending on whether initial curvature is in the direction of curling $(+)$ or in its opposite direction $(-)$.

Equations (6) and (1) can now be used in conjunction with Equation (12) to express flexural moment, M, as a function of recovery strain difference $(\epsilon_a - \epsilon_b)$:

$$M = (\epsilon_a - \epsilon_b)\{(t_a + t_b)Wt_at_bE_aE_b/2(t_aE_a + t_bE_b)\} \tag{13}$$

It is convenient to express the recovery strain difference $(\epsilon_a - \epsilon_b)$ as a function of the force applied externally to the composite strip during processing or usage. For this purpose we assume that the strains in the backing and the adhesive layer are the same and equal to ϵ initially due to the application of force, F. Consequently, simple force balance yields:

$$F = \epsilon W(E_at_a + E_bt_b) \tag{14}$$

Due to the highly viscous nature of the pressure sensitive adhesives commonly used, we also assume that the adhesive strains are largely unrecoverable or plastic in nature so that we can write

$$(\epsilon_a - \epsilon_b) = [1 - (E_a/E_b)]\epsilon \tag{15}$$

Note that Equation (15) allows some recovery in the adhesive layer even though for most pressure sensitive tapes the ratio (E_a/E_b) is in the order of 10^{-2}. The physical implication of Equation (15) is the following: The difference in the recovery strains for the adhesive and the backing is nearly equal (99%) to the strain, ϵ, induced by an initial tension force, F, applied to the composite strip prior to its curling. This implies that the backing recovers a large portion of the tensile strain while the adhesive retains a large portion of it. Substitution of Equation (14) in Equation (15) results in

$$(\epsilon_a - \epsilon_b) = [1 - (E_a/E_b)]\, F/W(E_a t_a + E_b t_b) \tag{16}$$

Further substitution of Equation (16) in Equation (13) results in our final relation for the flexural moment as

$$M = \{[1 - (E_a/E_b)](t_a + t_b)/2[(E_a t_a/E_b t_b) + (E_b t_b/E_a t_a) + 2]\}F \tag{17}$$

Equation (16) can similarly be substituted in Equation (12) to obtain a relation for the strip curvature $(1/\rho)$ as a function of the applied force F.

Discussion

We first note that the flexural moment, M, and the radius of curvature, ρ, are inversely proportional (Equation 6). In other words, if either one of them is an increasing function of a parameter, such as backing thickness t_b, etc., then the other would be expected to be a decreasing function of the same variable.

Inspection of Equation (17) reveals that the flexural moment decreases with increasing backing modulus and/or backing thickness while it increases with increasing adhesive thickness and/or adhesive modulus. These results can be seen more clearly in Figures 4 through 6 which show the plots of Equations (17) and (12) for the case of $E_b \cong 37$ ksi, $E_a \cong 377$ psi, $F = 0.6$ lb and $W = 3/4$ in.

Note that increasing radius of curvature, ρ, in Figures 4 through 6 indicate less flagging since in the limit tapes adhering to flat surfaces would have infinite radii of curvature. As expected, large radii of curvature correspond to smaller values of flexural moment.

A limited number of experiments performed using two different pressure sensitive adhesive tapes having different backing thicknesses and moduli provided data in reasonable agreement with Equation (12) when the strips were loaded at high rates initially. The agreement was closer for the strip with backing which had higher modulus but was thinner. The results of these experiments are shown in Tables I and II. In these experiments initial tension was induced by peeling off the release liner at 180° using an Instron testing machine under laboratory ambient conditions. The radii of curvature were measured using circular templates and also by making permanent impressions of the specimens. In general, the backing material for the composite tapes tested were various blends of vinyl with other polymers and plasticizers and coated with a latex adhesive.

Table I shows that for the pressure sensitive adhesive tape which had a backing with 37 ksi modulus and 1×10^{-3} in thickness, 50 in/min loading resulted in $\rho = 0.063$ in. The curl radius calculated based on the analysis of this paper is $\rho = 0.071$ in.

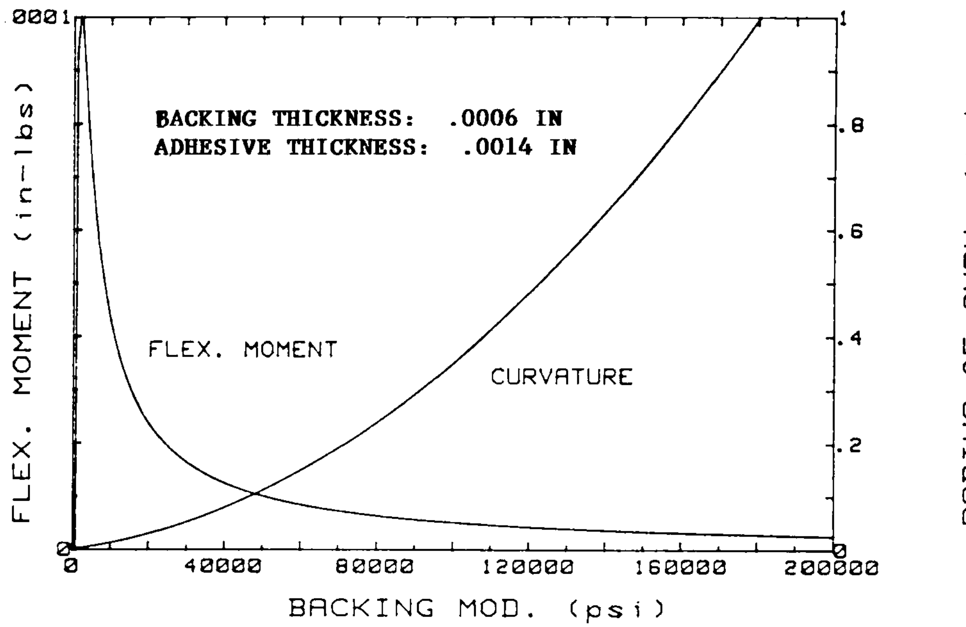

FIGURE 4 Variation of flexural moment and radius of curvature with backing modulus.

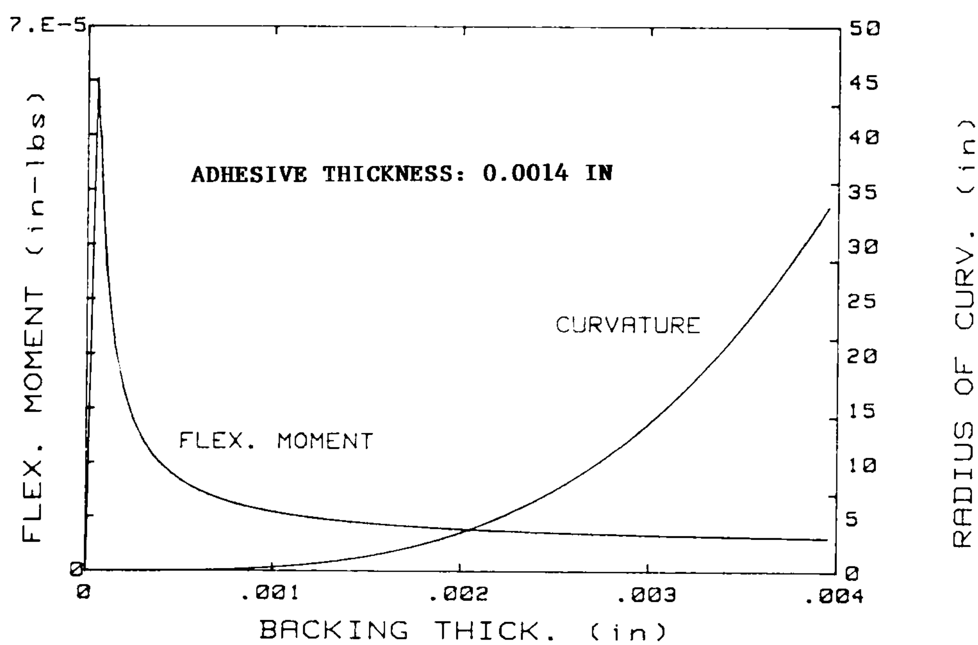

FIGURE 5 Variation of flexural moment and radius of curvature with backing thickness.

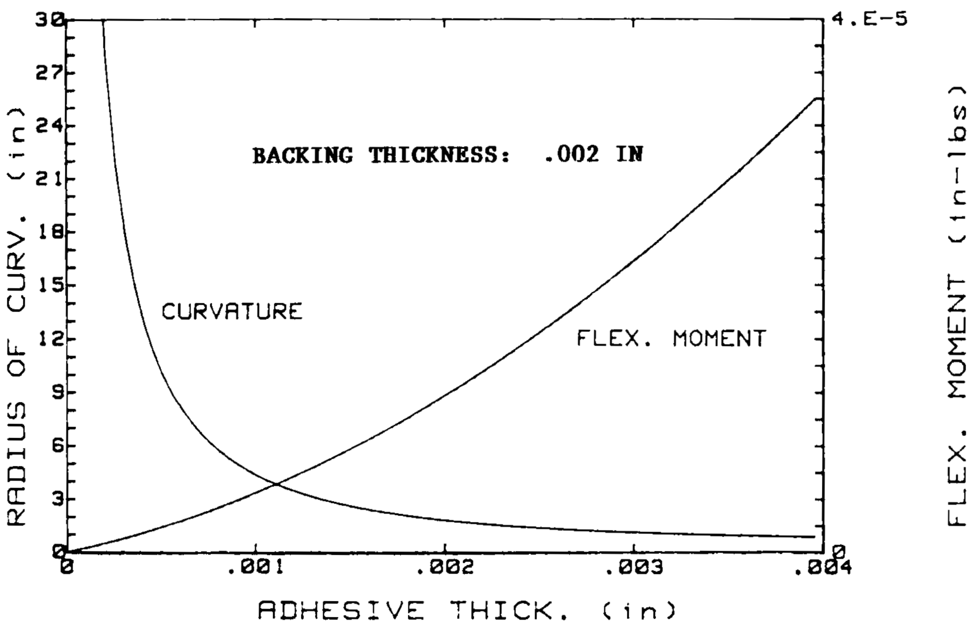

FIGURE 6 Variation of flexural moment and radius of curvature with adhesive thickness.

TABLE I
Experimental curling results—pressure sensitive tape no. 1
$t_b = 1 \times 10^{-3}$ in, $E_b = 37$ ksi
$t_a = 1 \times 10^{-3}$ in, $E_a = 0.37$ ksi

Number of specimens tested	Cross-head rate (in/min)	Max. peel force (lb)	Steady peel force (lb)	Curl radius (in)
2	0.2	0.53	0.35	0.055
2	50	0.61	0.46	0.063
				Calculated value = 0.071

TABLE II
Experimental curling results—pressure sensitive tape no. 2
$t_b = 3.5 \times 10^{-3}$ in, $E_b = 14$ ksi
$t_a = 1.0 \times 10^{-3}$ in, $E_a = 0.37$ ksi

Number of specimens tested	Cross-head rate (in/min)	Max. peel force (lb)	Steady peel force (lb)	Curl radius (in)
1	0.02	0.18	0.11	0.188
3	0.2	0.57	0.38	0.5 → 0.328
4	1	0.78	0.53	0.26 → 0.281
3	5	0.71	0.54	0.305 → 0.367
2	50	0.77	0.76	0.344 → ∞
				Calculated value = 0.53

For another tape with a three and a half times thicker but lower modulus ($E_b = 14$ ksi) backing, initial stretching with 50 in/min loading rate resulted in 0.344 in or higher curl radii (Table II). For this case the calculated value is $\rho = 0.53$ in. With this particular adhesive tape initial curl radii values as high as 0.5 in were obtained when initially stretched at rates as low as 0.2 in/min (Table II).

II—Analysis Based on Viscoelastic Backing Behavior:

Examination of Tables I and II also reveal viscoelastic effects illustrated by a general increase in the curl radius with time (Table II cross-head rates 1 in/min and higher) and increased loading rate (Tables I and II). The author attributes the observed increases in the curl radius over time for specimens initially stretched with cross-head speeds of 1 in/min and higher (in Table II) to the relaxation of the backing material over time. As shown in Figure 7, typical backing materials made of thermoplastic polymers are expected to exhibit relaxation behavior. At low levels of loading such as this case, this behavior can be approximated with the use of a linear viscoelastic model such as the Maxwell model. The relaxation behavior of the backing material can be included in the present analysis by expressing the flexural moment, which is proportional to the stress, as a function of time with the use of the Maxwell model. Consequently, Equation (17) can be written as

$$M(t) = F\{[1-(E_a/E_b)](t_a+t_b)/2[(E_at_a/E_bt_b)+(E_bt_b/E_at_a)+2]\}\{\exp(-t/T)\} \quad (18)$$

where t represents time and T is the relaxation time for the backing material which can be obtained by curve-fitting the Maxwell model's relaxation equation to the backing materials relaxation data such as is shown in Figure 7. Note that the use of a time-dependent equation for the relaxation time, T(t), may be necessary to approximate possible continuous change in the relaxation time as would be represented by a Maxwell chain.[4]

The reductions in the radius of curvature at lower cross-head rates, however, can be attributed to the lower values of the elastic modulus, E_b, of the viscoelastic backing material due to lower rates of loading. Such reductions in the elastic moduli of thermoplastic strips at lower loading rates is common and one example is illustrated in Table III. Examination of Figure 4 reveals that such reductions in the backing modulus results in smaller values for the radius of curvature and this is the behavior observed experimentally (Tables I, II). One can fit a semi-empirical equation such as Ludwik's Equation[5] to the available rate *versus* modulus data in the form

$$E_b(\dot{\varepsilon}) = (a_{T1})E_b' + (a_{T2})E_b''\text{Log}(\dot{\varepsilon}/\dot{\varepsilon}') \quad (19)$$

where $\dot{\varepsilon}$ is the strain rate, a_{T1} and a_{T2} are shift factors as functions of temperature and E', E'' and $\dot{\varepsilon}'$ are constants.

Equation (19) can now be substituted in Equation (18) to include the effects of rate on the flexural moment. Note that a similar relation can also be obtained for the curl radius of curvature as a function of rate, temperature and time by utilizing Equation (6).

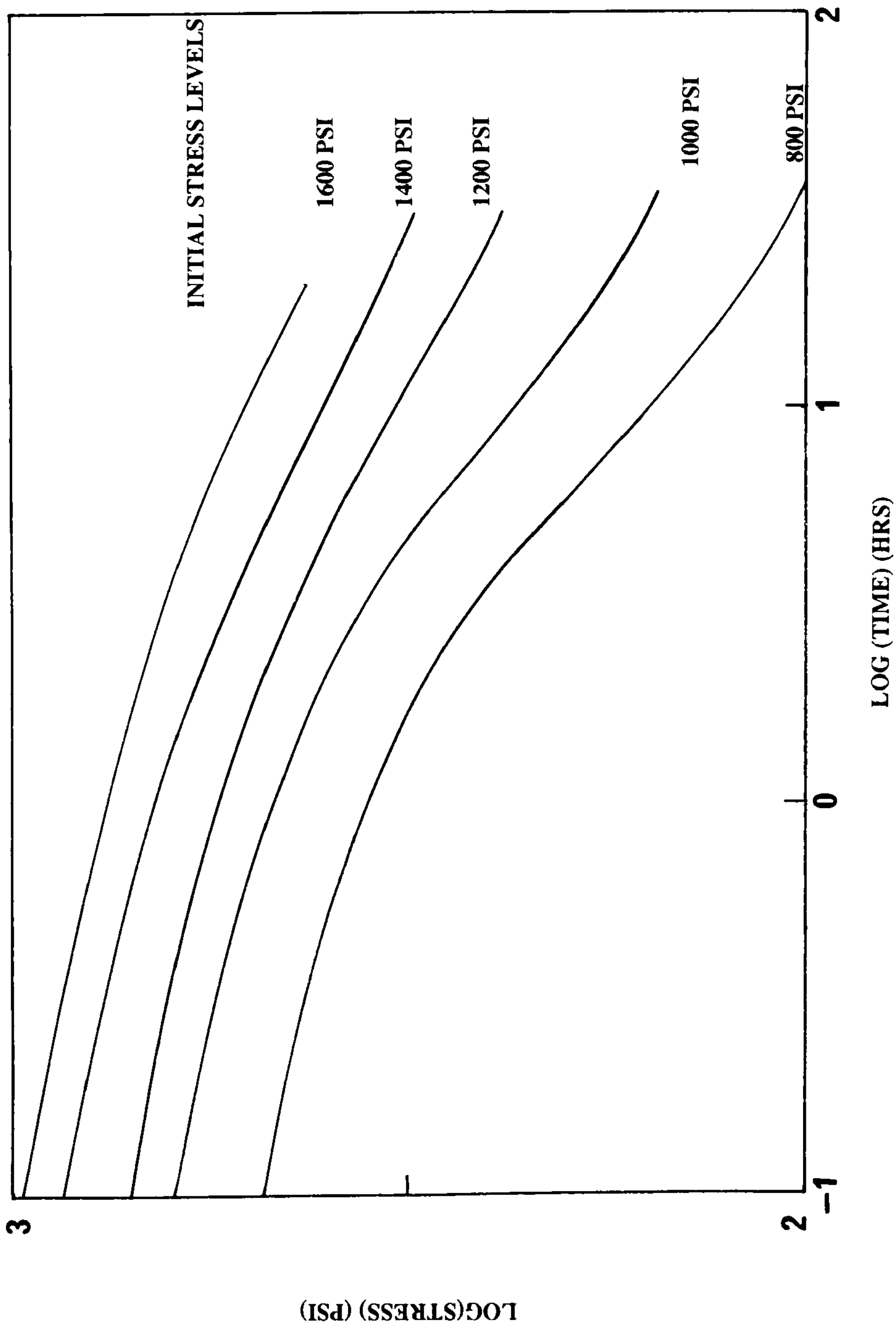

FIGURE 7 Relaxation data for a typical backing strip.

TABLE III
Variation of elastic properties with strain-rate for a typical polymer backing material

Cross-head rate (in/min)	Initial elastic strain rate (%/min)	Modulus of elasticity (ksi)	2.5% strain yield limit (psi)
0.5	17.2	33.2	930
2	67.2	38.8	1000
5	182	39	1070
10	378	39.5	1130

GENERAL EQUILIBRIUM CONDITION FOR PRESSURE SENSITIVE ADHESIVE STRIPS BONDED TO CURVED SURFACES

Given an equivalent stiffness, EI, for the composite strip, the overall moment equilibrium for an adhesive strip initially stretched and then bonded to a curved surface can be written as

$$M = EI/\rho = \int (F_a)dl + \int (\sigma_a)Wydy \tag{20}$$

where F_a is the adhesion force between the substrate surface and the composite strip and, σ_a is the thickness-dependent stress created in the viscoelastically deformed adhesive layer due to the presence of curvature. Note that Equation (20) represents a general case of bonding to a curved substrate surface and, consequently, the curvature term on its left-hand side should include the curvature of the surface also as explained earlier under Equation (12).

CONCLUSIONS

The present analysis shows that in pressure sensitive adhesive tapes recovery stresses and, consequently, the peeling moments are reduced by, i) increasing the thickness of the backing and/or, ii) increasing the rigidity (*i.e.* modulus of elasticity) of the backing. Increases in backing thickness should have a more pronounced effect. Furthermore, the flexural moment decreases as the adhesive thickness and/or adhesive modulus decreases. Obviously, if sufficient adhesion force is available between the strip and the substrate the presence of recovery stresses may be equilibrated. The presence of rate, time and temperature effects on the self-bending behavior can also be taken into account with the methodology presented.

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APPENDIX

Equivalent stiffness, EI, of the backing/adhesive composite strip, given by Equations (1) and (2) can be derived as follows:

Define elastic moduli ratio, n, as

$$n = E_a/E_b \tag{A1}$$

The cross section of the bimaterial strip is hypothetically transformed as shown in Figure A1 so that the width of the adhesive layer is now equal to nw while the width of the backing is maintained as w.

The centroid of the transformed section coincides with the neutral axis of the composite beam and is located at a distance y_c vertically from the bottom (free) surface of the adhesive layer:

$$y_c = \{t_b w[t_a + (t_b/2)] + t_a nw(t_a/2)\}/[nwt_a + wt_b] \tag{A2}$$

The area moment of inertia for the transformed composite section can be found by using the parallel axis theorem:

$$\bar{I}_x = (wt_b^3/12) + t_b w[(t_b/2) + t_a - y_c]^2 + (nwt_a^3/12) + nwt_a[y_c - (t_a/2)]^2 \tag{A3}$$

Substitution of Equation (A2) yields

$$\begin{aligned}\bar{I}_x = {} & (wt_b^3 + nwt_a^3)/12) + t_b w[(t_b/2) + t_a - (t_b^2 + 2t_b t_a + nt_a^2)/2(nt_a + t_b)]^2 \\ & + nwt_a[(t_b^2 + 2t_b t_a + nt_a^2)/2(nt_a + t_b) - (t_a/2)]^2\end{aligned} \tag{A4}$$

Equation (A4) is simplified using algebra:

$$\bar{I}_x = w[nt_b^3 t_a + t_b^4 + n^2 t_a^4 + nt_a^3 t_b + 3t_b^3 t_a n + 6t_b^2 t_a^2 n + 3t_b t_a^3 n]/12(nt_a + t_b) \tag{A5}$$

Substitution of Equation (A1) yields

$$\bar{I}_x = w[4E_a t_b t_a^3 + 6t_b^2 t_a^2 E_a + 4t_b^3 t_a E_a + t_b^4 E_b + (E_a^2/E_b)t_a^4]/12(E_a t_a + E_b t_b) \tag{A6}$$

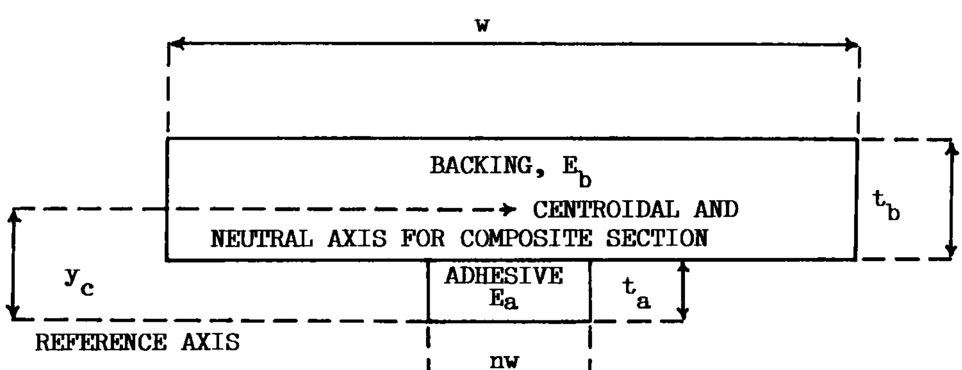

FIGURE A1 Transformed cross section of the bimaterial strip.

The equivalent stiffness for the composite beam is now obtained as

$$EI = E_b\bar{I}_x = w[4E_aE_bt_bt_a^3 + 6t_b^2t_a^2E_aE_b + 4t_b^3t_aE_aE_b + t_b^4E_b^2 + E_a^2t_a^4]/12(E_at_a + E_bt_b) \tag{A7}$$

Rearranging Equation (A7) we get

$$EI = \{wt_a^3t_bE_aE_b/12(E_at_a + E_bt_b)\}[4 + 6(t_b/t_a) + 4(t_b/t_a)^2 + (E_b/E_a)(t_bt_a)^3 + (E_a/E_b)(t_a/t_b)]. \tag{1}$$